

**AP Calculus BC**

Q1 Interim Assessment

Test Booklet 3

Scoring Guidelines

October 2017

School: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

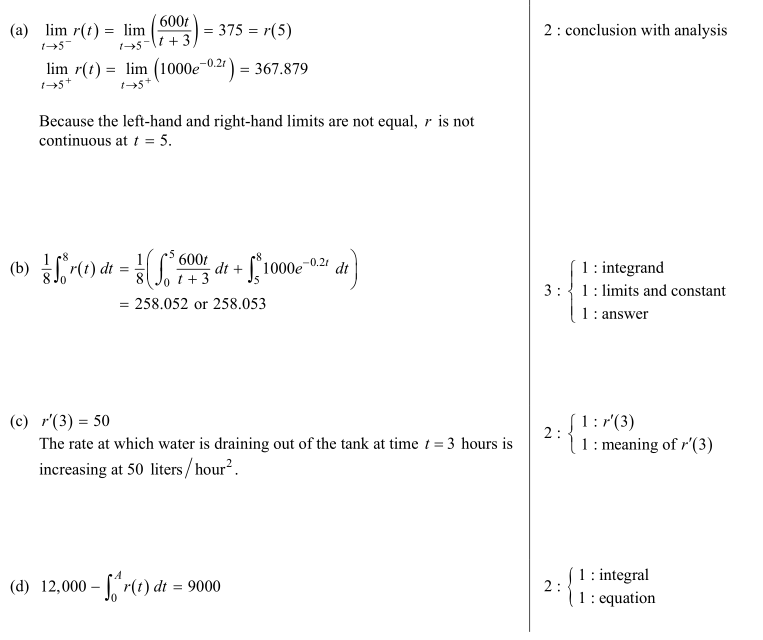
Student Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Teacher: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Period: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. A 12,000-liter tank of water is filled to capacity. At time , water begins to drain out of the tank at a rate modeled by , measured in liters per hour, where is given by the piecewise-defined function.

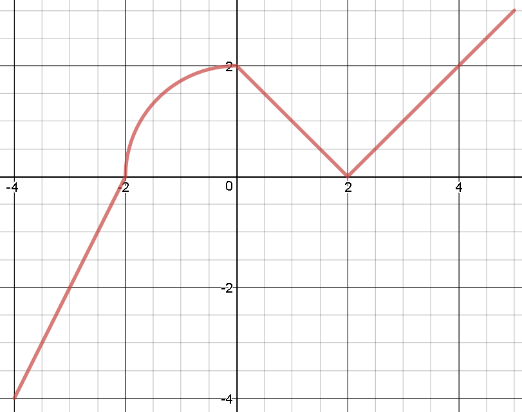
1. If continuous at ? Show the work that leads to your answer.
2. Find the average rate at which the water is draining from the tank between time and time hours.
3. Find . Using correct units, explain the meaning of that value in the context of this problem.
4. Write, but do not solve, an equation involving an integral to find the time when the amount of water in the tank is 9000 liters.



2. A particle moves along the -axis with velocity at time given by .

1. Find the average acceleration of the particle on the interval .
2. Find the instantaneous acceleration of the particle at time .
3. Is the speed of the particle increasing at time ? Give a reason for your answer.
4. Find all values of at which the particle changes direction. Justify your answer.

|  |  |
| --- | --- |
|  |  |
| is the instantaneous acceleration (rate of change) in  at . |  |
| Speed is increasing at  since  and . |  |
| 1. when , so .   for  and  for .  Therefore, the particle changes direction at . |  |



Graph of

3. Let be the continuous function defined on , whose graph, consisting of a quarter circle centered at and three line segments, is given above. Let be the function given by .

1. Find and , or state that the value does not exist.
2. For , find all values of for which the graph of has a point of inflection. Explain your reasoning.
3. Find the interval(s) on which is both increasing and concave down. Justify your answer.
4. Find the absolute maximum value of on the closed interval . Show the work that leads to your answer.

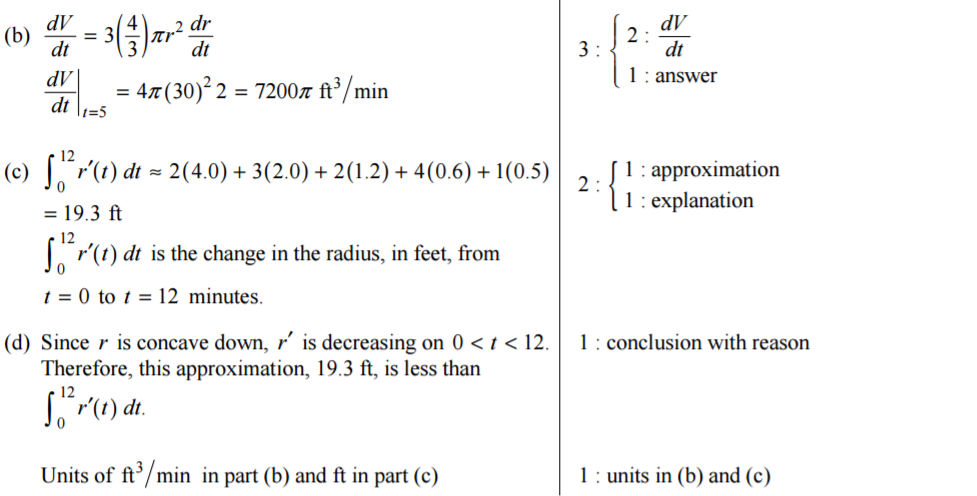
|  |  |
| --- | --- |
|  | 3 |
| 1. has a point of inflection at because changes sign at those -values. |  |
| 1. is increasing and concave down on because is positive and decreasing on this interval. |  |
| 1. The absolute maximum value of is  |  |  | | --- | --- | |  |  | | -4 | 4 | | -2 | 0 | | 2 |  | | 5 |  | | 2 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| (minutes) | 0 | 2 | 5 | 7 | 11 | 12 |
| (feet per minute) | 5.7 | 4.0 | 2.0 | 1.2 | 0.6 | 0.5 |

4. The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function of time , where is measured in minutes. For , the graph of is concave down. The table above gives selected values of the rate of change, , of the radius of the balloon over the time interval . The radius of the balloon is 30 feet when .

1. Approximate the value of . Indicate units of measure.
2. Find the rate of change of the volume of the balloon with respect to time when . Indicate units of measure.
3. Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate .
4. Is your approximation in part (c) greater than or less than ? Give a reason for your answer.

|  |  |
| --- | --- |
|  |  |



**^^^^ AWARDED IN QUESTION (C)** \_\_\_\_\_

5. Consider the curve known as the Folium of Descartes, given by the equation .

1. Show that .
2. Find the coordinate(s) of all points on the curve for which the tangent line is vertical.
3. Find the equation of the line tangent to the curve at the point .
4. Let and be functions of time that are related by the equation . At time , the values of and . Find the value of at time .

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6. Let be a differentiable function for which and whose derivative is given by the equation for all .

1. Find the *x*–coordinate of the critical point of *f*. Determine whether this point is a relative minimum, a relative maximum, or neither for the function *f*.
2. The graph of the function *f* has exactly one point of inflection. Find the *x*–coordinate of this point.
3. Find the equation of the tangent line to the curve at the point where . Use the tangent line to approximate the value of .
4. Evaluate .

|  |  |
| --- | --- |
| 1. at   changes from positive to negative at .  Thus, *f* has a relative maximum at . |  |
| at  changes from positive to negative at . Thus, the graph of *f* has a point of inflection when . |  |
|  |  |
| 1. by L’Hopital’s Rule   Answer: |  |